

Thermodynamical picture of the interacting holographic dark energy model

Dedicated to the 50 year Jubilee of Professor Sergei D. Odintsov

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Professor Sergei D. Odintsov has been active in various fields of theoretical physics, for this volume honouring his contribution to theoretical physics, I have chosen a work representative of our common interests.

Abstract

In the present paper, we provide a thermodynamical interpretation for the holographic dark energy model in a non-flat universe. For this case, the characteristic length is no more the radius of the event horizon (R_E) but the event horizon radius as measured from the sphere of the horizon (L). Furthermore, when interaction between the dark components of the holographic dark energy model in the non-flat universe is present its thermodynamical interpretation changes by a stable thermal fluctuation. A relation between the interaction term of the dark components and this thermal fluctuation is obtained.

¹Some parts of work done in collaboration with E. C. Vagenas

1 Introduction

Nowadays it is strongly believed that the universe is experiencing an accelerated expansion, and this is supported by many cosmological observations, such as SNe Ia [1], WMAP [2], SDSS [3] and X-ray [4]. These observations suggest that the universe is dominated by dark energy with negative pressure, which provides the dynamical mechanism of the accelerating expansion of the universe. Although the nature and origin of dark energy could perhaps be understood by a fundamental underlying theory unknown up to now, physicists can still propose some paradigms to describe it. In this direction we can consider theories of modified gravity [5], or field models of dark energy. The field models that have been discussed widely in the literature consider a cosmological constant [6], a canonical scalar field (quintessence) [7], a phantom field, that is a scalar field with a negative sign of the kinetic term [8, 9], or the combination of quintessence and phantom in a unified model named quintom [10]. The quintom paradigm intends to describe the crossing of the dark-energy equation-of-state parameter w_Λ through the phantom divide -1 [11], since in quintessence and phantom models the perturbations could be unstable as w_Λ approaches it [12].

In addition, many theoretical studies are devoted to understand and shed light on dark energy, within the string theory framework. The Kachru-Kallosh-Linde-Trivedi model [13] is a typical example, which tries to construct metastable de Sitter vacua in the light of type IIB string theory. Despite the lack of a quantum theory of gravity, we can still make some attempts to probe the nature of dark energy according to some principles of quantum gravity. An interesting attempt in this direction is the so-called “holographic dark energy” proposal [14, 15, 16, 17]. Such a paradigm has been constructed in the light of holographic principle of quantum gravity [18], and thus it presents some interesting features of an underlying theory of dark energy. Furthermore, it may simultaneously provide a solution to the coincidence problem, i.e why matter and dark energy densities are comparable today although they obey completely different equations of motion [16]. The holographic dark energy model has been extended to include the spatial curvature contribution [19] and it has been generalized in the braneworld framework [20]. Lastly, it has been tested and constrained by various astronomical observations [21].

In the present paper we study the thermodynamical interpretation of the interacting holographic dark energy model for a universe enveloped by the event horizon measured from the sphere of the horizon named L . We extend the thermodynamical picture in the case where there is an interaction term between the dark components of the HDE model. An expression for the interaction term in terms of a thermal fluctuation is given. In the limiting case of flat universe, we obtain the results derived in [22].

2 Interacting holographic dark energy density

In this section we obtain the equation of state for the holographic energy density when there is an interaction between holographic energy density ρ_Λ and a Cold Dark Matter(CDM) with $w_m = 0$. The continuity equations for dark energy and CDM are

$$\dot{\rho}_\Lambda + 3H(1+w_\Lambda)\rho_\Lambda = -Q, \quad (1)$$

$$\dot{\rho}_m + 3H\rho_m = Q. \quad (2)$$

The interaction is given by the quantity $Q = \Gamma\rho_\Lambda$. This is a decaying of the holographic energy component into CDM with the decay rate Γ . the quantity Q expresses the interaction between the dark components. The interaction term Q should be positive, i.e. $Q > 0$, which means that there is an energy transfer from the dark energy to dark matter. The positivity of the interaction term ensures that the second law of thermodynamics is fulfilled [23]. At this point, it should be stressed that the continuity equations imply that the interaction term should be a function of a quantity with units of inverse of time (a first and natural choice can be the Hubble factor H) multiplied with the energy density. Therefore, the interaction term could be in any of the following forms: (i) $Q \propto H\rho_X$ [24, 23], (ii) $Q \propto H\rho_m$ [25], or (iii) $Q \propto H(\rho_X + \rho_m)$ [26]. The freedom of choosing the specific form of the interaction term Q stems from our incognizance of the origin and nature of dark energy as well as dark matter. Moreover, a microphysical model describing the interaction between the dark components of the universe is not available nowadays. Taking a ratio of two energy densities as $r = \rho_m/\rho_\Lambda$, the above equations lead to

$$\dot{r} = 3Hr\left[w_\Lambda + \frac{1+r}{r}\frac{\Gamma}{3H}\right] \quad (3)$$

Following Ref.[27], if we define

$$w_\Lambda^{\text{eff}} = w_\Lambda + \frac{\Gamma}{3H}, \quad w_m^{\text{eff}} = -\frac{1}{r}\frac{\Gamma}{3H}. \quad (4)$$

Then, the continuity equations can be written in their standard form

$$\dot{\rho}_\Lambda + 3H(1+w_\Lambda^{\text{eff}})\rho_\Lambda = 0, \quad (5)$$

$$\dot{\rho}_m + 3H(1+w_m^{\text{eff}})\rho_m = 0 \quad (6)$$

We consider the non-flat Friedmann-Robertson-Walker universe with line element

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2d\Omega^2\right). \quad (7)$$

where k denotes the curvature of space $k=0,1,-1$ for flat, closed and open universe respectively. A closed universe with a small positive curvature ($\Omega_k \sim 0.01$) is compatible with observations [28, 29]. We use the Friedmann equation to relate the curvature of the universe to the energy density. The first Friedmann equation is given by

$$H^2 + \frac{kc^2}{a^2} = \frac{1}{3M_p^2}\left[\rho_\Lambda + \rho_m\right]. \quad (8)$$

Define as usual

$$\Omega_m = \frac{\rho_m}{\rho_{cr}} = \frac{\rho_m}{3M_p^2H^2}, \quad \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{cr}} = \frac{\rho_\Lambda}{3M_p^2H^2}, \quad \Omega_k = \frac{kc^2}{a^2H^2} \quad (9)$$

Now we can rewrite the first Friedmann equation as

$$\Omega_m + \Omega_\Lambda = 1 + \Omega_k. \quad (10)$$

Using Eqs.(9,10) we obtain following relation for ratio of energy densities r as

$$r = \frac{1 + \Omega_k - \Omega_\Lambda}{\Omega_\Lambda} \quad (11)$$

In non-flat universe, our choice for holographic dark energy density is

$$\rho_\Lambda = 3c^2 M_p^2 L^{-2}. \quad (12)$$

As it was mentioned, c is a positive constant in holographic model of dark energy ($c \geq 1$) and the coefficient 3 is for convenient. L is defined as the following form:

$$L = ar(t), \quad (13)$$

here, a , is scale factor and $r(t)$ can be obtained from the following equation

$$\int_0^{r(t)} \frac{dr}{\sqrt{1 - kr^2}} = \int_t^\infty \frac{dt}{a} = \frac{R_h}{a}, \quad (14)$$

where R_h is event horizon. Therefore while R_h is the radial size of the event horizon measured in the r direction, L is the radius of the event horizon measured on the sphere of the horizon. For closed universe we have (same calculation is valid for open universe by transformation)

$$r(t) = \frac{1}{\sqrt{k}} \sin y. \quad (15)$$

where $y \equiv \sqrt{k}R_h/a$. Using definitions $\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{cr}}$ and $\rho_{cr} = 3M_p^2 H^2$, we get

$$HL = \frac{c}{\sqrt{\Omega_\Lambda}} \quad (16)$$

Now using Eqs.(13, 14, 15, 16), we obtain

$$\dot{L} = HL + a\dot{r}(t) = \frac{c}{\sqrt{\Omega_\Lambda}} - \cos y, \quad (17)$$

By considering the definition of holographic energy density ρ_Λ , and using Eqs.(16, 17) one can find:

$$\dot{\rho}_\Lambda = -2H(1 - \frac{\sqrt{\Omega_\Lambda}}{c} \cos y)\rho_\Lambda \quad (18)$$

Substitute this relation into Eq.(1) and using definition $Q = \Gamma\rho_\Lambda$, we obtain

$$w_\Lambda = -(\frac{1}{3} + \frac{2\sqrt{\Omega_\Lambda}}{3c} \cos y + \frac{\Gamma}{3H}). \quad (19)$$

Here as in Ref.[30], we choose the following relation for decay rate

$$\Gamma = 3b^2(1+r)H \quad (20)$$

with the coupling constant b^2 . Using Eq.(11), the above decay rate take following form

$$\Gamma = 3b^2H \frac{(1 + \Omega_k)}{\Omega_\Lambda} \quad (21)$$

Substitute this relation into Eq.(19), one finds the holographic energy equation of state

$$w_\Lambda = -\frac{1}{3} - \frac{2\sqrt{\Omega_\Lambda}}{3c} \cos y - \frac{b^2(1 + \Omega_k)}{\Omega_\Lambda}. \quad (22)$$

3 Thermodynamical interpretation of the interacting HDE model

Following [22] (see also [23]), the non-interacting HDE model in the non-flat universe as described above is thermodynamically interpreted as a state in thermodynamical equilibrium. According to the generalization of the black hole thermodynamics to the thermodynamics of cosmological models, we have taken the temperature of the event horizon to be $T_L = (1/2\pi L)$ which is actually the only temperature to handle in the system. If the fluid temperature of the cosmological model is set equal to the horizon temperature (T_L), then the system will be in equilibrium. Another possibility [31] is that the fluid temperature is proportional to the horizon temperature, i.e. for the fluid enveloped by the apparent horizon $T = eH/2\pi$ [32]. In general, the systems must interact for some length of time before they can attain thermal equilibrium. In the case at hand, the interaction certainly exists as any variation in the energy density and/or pressure of the fluid will automatically induce a modification of the horizon radius via Einstein's equations. Moreover, if $T \neq T_L$, then energy would spontaneously flow between the horizon and the fluid (or viceversa), something at variance with the FRW geometry [33]. Thus, when we consider the thermal equilibrium state of the universe, the temperature of the universe is associated with the horizon temperature. In this picture the equilibrium entropy of the holographic dark energy is connected with its energy and pressure through the first thermodynamical law

$$TdS_\Lambda = dE_\Lambda + p_\Lambda dV \quad (23)$$

where the volume is given as

$$V = \frac{4\pi}{3}L^3 , \quad (24)$$

the energy of the holographic dark energy is defined as

$$E_\Lambda = \rho_X V = 4\pi c^2 M_p^2 L \quad (25)$$

and the temperature of the event horizon is given as

$$T = \frac{1}{2\pi L^0} . \quad (26)$$

Substituting the aforesaid expressions for the volume, energy, and temperature in equation (23) for the case of the non-interacting HDE model, one obtains

$$dS_\Lambda^{(0)} = 8\pi^2 c^2 M_p^2 (1 + 3\omega_\Lambda^0) L^0 dL^0 \quad (27)$$

where the superscript (0) denotes that in this thermodynamical picture our universe is in a thermodynamical stable equilibrium.

In the interacting case, by substituting equation (18) in the conservation equation (1) for the dark energy component one obtains

$$1 + 3\omega_\Lambda = -2\frac{\sqrt{\Omega_\Lambda}}{c} \cos y - \frac{Q}{3H^3 M_p^2 \Omega_\Lambda} . \quad (28)$$

According to [22], the interacting HDE model in the non-flat universe as described above is not anymore thermodynamically interpreted as a state in thermodynamical equilibrium.

In this picture the effect of interaction between the dark components of the HDE model is thermodynamically interpreted as a small fluctuation around the thermal equilibrium. Therefore, the entropy of the interacting holographic dark energy is connected with its energy and pressure through the first thermodynamical law

$$TdS_\Lambda = dE_\Lambda + p_\Lambda dV \quad (29)$$

where now the entropy has been assigned an extra logarithmic correction [34]

$$S_\Lambda = S_\Lambda^{(0)} + S_\Lambda^{(1)} \quad (30)$$

where

$$S_\Lambda^{(1)} = -\frac{1}{2} \ln (CT^2) \quad (31)$$

and C is the heat capacity defined by

$$C = T \frac{\partial S_\Lambda^{(0)}}{\partial T} \quad (32)$$

and using equations (27), (26), is given as

$$C = -8\pi^2 c^2 M_p^2 (L^0)^2 (1 + 3\omega_\Lambda^0) \quad (33)$$

$$= 16\pi^2 c M_p^2 (L^0)^2 \sqrt{\Omega_\Lambda^0} \cos y . \quad (34)$$

Substituting the expressions for the volume, energy, and temperature (it is noteworthy that these quantities depend now on L and not on L^0 since there is interaction among the dark components) in equation (29) for the case of the interacting HDE model, one obtains

$$dS_\Lambda = 8\pi^2 c^2 M_p^2 (1 + 3\omega_\Lambda) L dL \quad (35)$$

and thus one gets

$$1 + 3\omega_\Lambda = \frac{1}{8\pi^2 c^2 M_p^2 L} \frac{dS_\Lambda}{dL} \quad (36)$$

$$= \frac{1}{8\pi^2 c^2 M_p^2 L} \left[\frac{dS_\Lambda^{(0)}}{dL} + \frac{dS_\Lambda^{(1)}}{dL} \right] \quad (37)$$

$$= -2 \left(\frac{\sqrt{\Omega_\Lambda^0}}{c} \cos y \right) \frac{L^0}{L} \frac{dL^0}{dL} + \frac{1}{8\pi^2 c^2 M_p^2 L} \frac{dS_\Lambda^{(1)}}{dL} \quad (38)$$

where the last term concerning the logarithmic correction can be computed using expressions (31) and (34)

$$\frac{dS_\Lambda^{(1)}}{dL} = -\frac{H}{\left(\frac{c}{\sqrt{\Omega_\Lambda^0}} - \cos y \right)} \left[\frac{(\Omega_\Lambda^0)'}{4\Omega_\Lambda^0} + y \tan y \right] \quad (39)$$

with the prime (') to denote differentiation with respect to $\ln a$.

Therefore, by equating the expressions (28) and (38) for the equation of state parameter of the holographic dark energy evaluated on cosmological and thermodynamical grounds respectively, one gets an expression for the interaction term

$$\frac{Q}{9H^3M_p^2} = \frac{\Omega_\Lambda}{3} \left[-\frac{2\sqrt{\Omega_\Lambda}}{c} \cos y + \left(\frac{2\sqrt{\Omega_\Lambda}}{c} \cos y \right) \frac{L^0}{L} \frac{dL^0}{dL} \right] - \frac{1}{8\pi^2 c^2 M_p^2 L} \frac{\Omega_X}{3} \frac{dS_\Lambda^{(1)}}{dL}. \quad (40)$$

It is noteworthy that in the limiting case of flat universe, i.e. $k = 0$, we obtain exactly the result derived in [22] when one replaces L^0 and L with R_E^0 and R_E , respectively.

4 Conclusions

It is of interest to remark that in the literature, the different scenarios of DE has never been studied via considering special similar horizon, as in [31] the apparent horizon, $1/H$, determines our universe. For flat universe the convenient horizon looks to be event horizon, while in non flat universe we define L because of the problems that arise if we consider event horizon or particle horizon (these problems arise if we consider them as the system's IR cut-off). Thus it looks that we need to define a horizon that satisfies all of our accepted principles; in [35] a linear combination of event and apparent horizon, as IR cut-off has been considered. In present paper, we studied L , as the horizon measured from the sphere of the horizon as system's IR cut-off. In the present paper, we have provided a thermodynamical interpretation for the HDE model in a non-flat universe. We utilized the horizon's radius L measured from the sphere of the horizon as the system's IR cut-off. We investigated the thermodynamical picture of the interacting HDE model for a non-flat universe enveloped by this horizon. The non-interacting HDE model in a non-flat universe was thermodynamically interpreted as a thermal equilibrium state. When an interaction between the dark components of the HDE model in the non-flat universe was introduced the thermodynamical interpretation of the HDE model changed. The thermal equilibrium state was perturbed by a stable thermal fluctuation which was now the thermodynamical interpretation of the interaction. Finally, we have derived an expression that connects this interaction term of the dark components of the interacting HDE model in a non-flat universe with the aforesaid thermal fluctuation.

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